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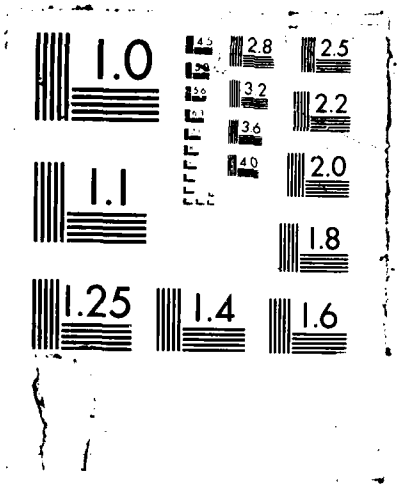
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MATHEMATICAL MODELS of SOUND WAVES in FLUIDS

by Garrett Birkhoff

FINAL TECHNICAL REPORT on CONTRACT N00014-85-0-0466

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The bulk of the research performed under this contract has dealt with mathematical problems of numerical ocean acoustics of the kind described in Ref. 1, pp. 643-54. These concerned the propagation of sound waves in (generally inhomogeneous) elastic fluids, with special reference to the consistency of the elastic fluid model with 'ray theory' (Fermat-Huygens), in predicting reflection, refraction, and diffraction.

I have written up most of the fruits of this research for publication in Refs. 2-4, of which Ref. 2 has appeared (reprint enclosed). A copy of Ref. 3 has been retyped at the Naval Underwater Systems Center for photographic reproduction. Typescripts of Refs. 4 and 5 have been sent to the editors of the Proceedings in which they will appear. Much of Ref. 4 is a less detailed but more polished exposition of results presented in Refs. 2 and 3. These results include the following:

A. Formulation of a precise mathematical definition of the concept of an inhomogeneous elastic fluid (Ref. 3, Sec. 9 and Appendix C; cf. Ref. 2, Secs. 10 and 15).

B. Derivation plane of exact second-order linear differential equations for waves in inhomogeneous Chaplygin fluids. (Ref. 3, Secs. 10 and 14; Ref. 1, Sec. 6).

C. The conclusion that, in more than one dimension, sound waves in such fluids will usually generate first-order vorticity (Ref. 3, Secs. 9 and 11). Thus the usual assumption of the existence of a velocity potential is not justified, even if there is no viscosity.

D. The tentative conclusion that, nevertheless, Fermat's and Huygens' Principles are asymptotically valid for very short waves in arbitrary elastic fluids (Ref. 4, Sec. 6).

A-1

E. Clarification of the physical significance of the Sommerfeld-Runge eikonal function for sound waves in fluids (Ref. 2, Sec. 10; Ref. 3, Sec. 5; Ref. 4, Sec. 6).

Namely, each "eikonal surface" on which the Sommerfeld-Runge eikonal function assumes a constant value is asymptotically an isobaric surface of constant pressure at all times, associated with "simple" sound waves (Ref. 1, p. 646) in which the fluid is constrained worklessly to move along acoustic rays.

The preceding results hold in static fluids. In moving fluids (Ref. 3, Secs. 7-8), although ray theory can still be applied, the use of variational principles involves the concept of a Finsler space, and very unfamiliar geometrical ideas.

The elastic fluid models discussed above all conserve mechanical energy; they do not predict any absorption or dispersion. I have made a critical analysis, in historical perspective, of several standard treatises concerned with models intended to explain these related phenomena.** My impressions, reported in Ref. 3 (Secs. 3-5 and Appendix A) and Ref. 4 (Secs. 7-11), are summarized in the Appendix to the present Report. I have correlated them with the discussion of absorption and dispersion (primarily in gases) on pp. 549-562 of Allan Pierce's admirable book Acoustics.

My main conclusion is that the standard modern explanation in terms of relaxation times, although sixty years old, has not yet been

** I took as "standard" the books by A.B. Wood and by Herzfeld and Litovitz; the review article by Markham, Feyer, and Lindsay; and articles by Lighthill and by Truesdell, all cited by Pierce; and the article by E.V. Ryan in the 1957 American Institute of Physics Handbook.

substantiated (especially in liquids) by clear answers to many basic questions. These include the following:

F. To what extent is the absorption of sound per wave length, $\alpha\lambda$, in air, CO_2, and other dilute gases determined by the absolute temperature, T, and the ration f/p of the frequency to the pressure?

G. To what extent are contributions to α from different causes demonstrably additive, in gases and in liquids?

H. How well can one predict the locations, breadths, and heights of the two graphs drawn in Fig. 10-12 of Pierce? How well are these correlated experimentally, in gases and in liquids?

I. How are the bulk viscosities of liquids and gases best defined and measured? The old book by A. Barr, A Manual of Viscometry, on shear viscosity, sets a good example.

Computational acoustics. When this contract was set up, I planned to polish my 1983 soft-cover monograph Numerical Fluid Dynamics for publication in hard-cover form, including in it new material. A paper submitted to SIAM Review on "Difference methods for solving convection-diffusion equations," coauthered by E.C. Garland, Jr., and R.E. Lynch, was written with this in mind. So was the visit to Harvard by Prof. Lynch in the summer of 1986, during which we discussed improving difference approximations to Neumann and mixed boundary conditions. Since then, Prof. Lynch has incorporated these into the Purdue ELLPACK package, and written two papers on the subject.

I planned to do extensive rewriting in connection with a seminar on Numerical Fluid Dynamics which I was scheduled to lead at the Naval Postgraduate School last winter. However, this project progressed slowly, and after a heart attack made me give up this semester in December, I abandoned it. Instead, I spent time preparing Ref. 5,

over half of which is devoted to publicizing the major role played by navy-sponsored research in the crucial early development of scientific computing. This invited paper will appear in a volume which, I believe, will be widely read.

Appendix. Absorption and Dispersion

Already by 1860, general mathematical theories of viscosity (Stokes) and heat conduction (Fourier) had been constructed. Both of these theories predict that sound waves in fluids should be subjected to absorption and dispersion, and that the rate of internal sound absorption, α, should be proportional to f^2, the square of the frequency. Actually, the two theories can be incorporated into a more elaborate theory of homogeneous viscous thermoelastic fluids. This theory involves three parameters besides those which characterize elastic fluids: the thermal conductivity κ, the shear viscosity μ, and the bulk viscosity μ'.

Until 1925, this theory seemed adequate, partly because the internal absorption and dispersion of sound in air and water, on a laboratory scale (< 10m), are almost imperceptible over the audible range (roughly 25 Hz-5 kHz). The effects of κ and μ' were simply added to those of μ, giving

$$\alpha_{c\ell} = (C_\mu + C_\kappa + C_{\mu'}) f^2 \quad , \tag{1}$$

where typically $C_\mu < C_\kappa + C_{\mu'} < 2C_\mu$ in gases. Moreover, the concept of a viscous thermoelastic fluid was believed to be confirmed by the Maxwell-Boltzmann kinetic theory of gases, because this theory explained plausibly variations in μ and κ with changes in the temperature T and pressure p. It also predicts that, in gases at given temperature, should be a function of f/p alone.

However, beginning around 1925, observations of ultrasoni

waves in the range 5 kHz-1 mHz revealed rates of internal sound absorption in fluids of $10\alpha_{c\ell}$ - $100\alpha_{c\ell}$ or even more. These are generally attributed to molecular relaxation effects, associated with "relaxation times" τ_m required to transfer "rotational" to "translational" energy. In gases, this concept can be rationalized in terms of Maxwell-Boltzmann mass-spring models of molecules; moreover, kinetic theory suggests that $\lambda\alpha$ should be a single-valued function of the ratio f/p of the sound wave frequency to the gas pressure. From 1925 to 1940, discoveries of major deviations from (1) in gases and liquids were correlated with developments in theoretical molecular physics, which was being revolutionized by quantum mechanics.

After World War II, intense interest in shock waves helped to stimulate extensive further studies of the absorption of mechanical energy in 80 or more chemically pure fluids. Liquids were classified into qualitatively different types, as regards their sound absorption properties. In addition, important effects of impurities and sound absorption in mixtures (such as air) continued to be studied.

Bulk viscosity also received careful scrutiny for the first time after 1940. Thus G.I. Taylor (Scientific Papers, iv, #23) showed that air bubbles in water can, theoretically, make $\mu' = 6700\mu$. Clifford Truesdell made the first rigorous analysis of the theoretical dependence of α on the dimensionless ratios μ'/μ and $Pr = c_p\mu/k$ in a general viscous thermoelastic fluid, and of theoretical deviations from (1). His critique of procedures used to fit "relaxation

times" to empirical data has never been clearly answered. It would be desirable to have a clearer idea of the magnitude of Truesdell's corections to formula (1), which is still believed to hold (with $\mu' = 0$) for inert gases like He.

Of even greater interest is the empirical evidence for Pierce's formula (10-8.11),

$$\alpha(f) = \alpha_{cl}(f) + \sum_{\nu} \alpha_{\nu}(f) \quad , \tag{2}$$

where different α_{ν} dominate over different ranges of f, and his analogous formula (10-8.16) for dispersion. It is not clear for which gases and liquids these formulas have been proven experimentally to hold, with what precision over what ranges of f and states (p,T). Likewise, for which pure and impure fluids have the parameters $(\alpha_{\nu}\lambda)_m$ and τ_{ν} which occur in these formulas been deduced from general basic formulas of quantum mechanics, and with what precision?

Until such questions (which are related to Questions F-1 stated in the body of this report) are answered, I think that the status of 'relaxation time' rationalizations of absorption and dispersion phenomena resembles that of 'classical' rationalizations as of 1900. Although highly plausible, their range of validity (especially for liquids) is unclear!

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